

## CHANGE ORGANIZATION IN RANDOM TIME

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**Abstract:** The work is an attempt to answer a question concerning of amount expenditures changes to be incurred during reorganization (system changes) of management model for case with random horizon (e.g. a random number of factors to be take into account during the system immunization). Also the differences of expenditures for cases with fixed and random horizons (number changes) are shown.

**Key Words:** change control problem, follower problem, random horizon, linear quadratic control

**JEL Classification:** R11, P10, M10

### 1. INTRODUCTION

To perform specific tasks the enterprises, companies, organizations must adapt to conditions in which they operate, work, perform their tasks. Unfortunately, the environment is not stable, constant, permanent, and therefore it is necessary to make the changes. The word change means the accommodate their tasks and functions to existing conditions as well as immunization (protection, security) its management system to unfavorable internal and external factors. The actions of organization, companies should be more efficient, effective, perfect, favorable, then the changes should be made by a specific pattern, model. The creative and innovative thinking and cybernetic view of chief executives (directors, administrations) allows to model the behavior of organization, enterprise, system in turbulent environment and helps to make the appropriate changes. This allows lead the organization into an idealized image, model, which works best in existing circumstances in accordance with chief executives (board of directors). The main objective is accommodation (adaptation) of organization, but also to take into account the costs of these changes.

The controls (changes) are in some sense a fight against disorder, an attempt of bringing the economic, technical, phisical systems to suitable (required) state. In the area of Management Sciences or so-called Organizational Change Management the changes are introduced and managed by some people as reactions to variable environment. Sometimes the changes are reactions resulting from discussions about how organizations should be function. Unfortunately, there is no general and systematic approach for investigation of the processes of such changes. Instead, we have many heuristic approaches to change making (modeling, designing) see e.g. Carillo, Gaimon 2000, Carr, Hancock 2006, Carr 2002, Ford 2002, Jonnergard, Karreman, Svenson 2004, Melkonian 2004-5, Reverdy 2006, Yu, Ming 2008). In Banek, Kozłowski 2010 the authors proposes the change organization through analysis, similarity and comparison a stochastic Monotone Follower Problem (MFP) as a natural candidate to model the Change Control Problem. This work is a continuation of MFP and show to make changes for random horizon.

### 2. THE FOLLOWER PROBLEM IN $R^n$

Let us introduce two systems Evader and Follower as in the paper Banek, Kozłowski 2010. Let  $(\Omega, F, P)$  be a complete probability space where the random variables  $\xi_0, w_1, \dots, w_N, \theta_1, \dots, \theta_N$  are defined. For  $f: R^n \times R^m \rightarrow R^n$  some measurable function called the dynamic function. The stochastic system

$$\xi_{i+1} = f(\xi_i, \varepsilon \theta_{i+1} w_{i+1}) \quad (1)$$

is called the Evader (E), where  $\varepsilon > 0$ , and the product  $\varepsilon \theta_{i+1} w_{i+1}$  models stochastic disturbances occurring in the system. Since  $\theta_i = 1$  or  $0$ , the disturbance  $\varepsilon w_i$  occurs in time  $i$  or not. By allowing  $\varepsilon$  and  $p$  to have bigger or smaller values we can model the intensity of the random disturbances affecting the movement of Evader. This system represents an image, view of state organization. Sometimes the above system is called a leader (benchmarking), which must be imitated, followed, chased.

Another system is called the Follower (F) and described by the iterative scheme

$$x_{i+1} = g(x_i, u_i) \quad (2)$$

where  $g: R^n \times R^m \rightarrow R^n$  some measurable function. Here, by  $u_i$  we denote the control action in the time  $i$ . In this sense, the Follower represents the state our organization. Let the random variable  $\tau$  means the horizon of changes (the time in which we must make the changes).

$$P(\tau = i) = p_i \text{ for } i = 0, 1, 2, \dots, N,$$

$$\text{where } 0 \leq p_i \leq 1 \text{ and } \sum_{i=0}^N p_i = 1 \quad (3)$$

and  $z: R^n \times R^m \times R^n \rightarrow R_+$ ,  $h: R^n \times R^n \rightarrow R_+$  be a some Borel measurable and bounded below function, which represent losses and heredity, suitable. The performance criterion is

$$J(u) = E \left[ \sum_{j=0}^{\tau-1} z(x_j, u_j, \xi_{j+1}) + h(x_\tau, \xi_\tau) \right] \quad (4)$$

where  $E$  denotes expectation with respect to the measure  $P$ . The aim of the Follower is to find

$$\min_{u \in U} J(u) \quad (5)$$

where  $U = \{u_i = v_i(\xi_0, \dots, \xi_i, x_0, \dots, x_i); i = 0, \dots, N-1\}$ . Using the total probability formula we transform the performance criterion (4) to next form

$$J(u) = E \left[ \sum_{j=0}^{N-1} P(\tau \geq j) z(x_j, u_j, \xi_{j+1}) + P(\tau = j) h(x_j, \xi_j) + P(\tau = N) h(x_N, \xi_N) \right] \quad (6)$$

### 3. THE CHANGE PROBLEM IN RANDOM TIME

Most of the management issues are modeled and considered for the cases when the horizon of changes are determined. But if we try to imitate some idealized image and try to adjust our structure then we have not always a fixed horizon changes. For example, creating a security system we must try to immunize them to various external factors which are introducing the system destabilizing. How many external factors should be considered? And do you take into account new and current? It is not always possible to identify all factors.

For tasks of organizational structure adapting to a picture (an image) we can find two cases with the random horizon. One of these cases consists the determining of stopping moment (we must determine the optimal moment at which we must necessary stop the changes, production, etc.). Quite often we have these problems in the life cycle of product (we must answer the question when to stop production and make the changes of product structure, production line, etc.) The second type of tasks describes a situation where we must adapt in random time our organization to the environment or we must reorganize same structures in random time. The paper Banek, Kozłowski 2010 answers the question how to make changes to faithfully mimic the Evader in a fixed time (to adjust our structure to the pattern), and the total cost changes as small as possible.

Indeed, the task from work Banek, Kozłowski 2010 can be modified and considered until moment  $[E\tau]$  equal the expected value of random horizon. And how to make changes (control system) if we cross, exceed the time  $[E\tau]$ ? And how to act (make changes) if it turns out that the horizon does not achieve the value  $[E\tau]$ . In this case the system (organization) changes have not been fully carried out and the system is far from the Evader (image, pattern). Building a model of the changes appropriate to the situation we avoid the above inconvenience. The solution of task with a random horizon independent of the system states shows how we must make the changes in such situations.

### 4. THE FOLLOWER PROBLEM IN RANDOM TIME

Let the Evader system is described by a fixed state (point)  $\xi \in R^n$ , but the Follower system at time  $j$  is described by linear equation

$$y_{j+1} = y_j - Bu_j + \sigma w_{j+1} \quad (7)$$

where  $w_1, w_2, \dots, w_N$  represent a disturbances and are modeled by a random vectors with normal distribution  $N(0, I)$ . We assume that  $w_1, w_2, \dots, w_N$  are independent. Let the cost of changes at time  $j$  has a quadratic form as  $a\|u_j\|^2$  and the heredity function is defined as a quadratic differences between Evader's state  $\xi$  and Follower's state  $y_j$ . The random variable  $\tau$  presents a horizon of control (changes)

and has a Bernoulli distribution with the probability of success  $0 \leq p \leq 1$ . The performance criterion is

$$\inf_{u \in U} E \left\{ \sum_{i=0}^{\tau-1} a\|u_i\|^2 + b\|y_\tau - a\|^2 \right\} \quad (8)$$

which we can decompose to next form

$$\inf_{u \in U} E \left\{ \sum_{i=0}^{N-1} (a_i\|u_i\|^2 + b_i\|y_i - a\|^2) + b_N\|y_N - a\|^2 \right\} \quad (9)$$

Where

$$a_i = a \sum_{k=i+1}^N \binom{N}{k} p^k (1-p)^{N-k} \text{ for } i = 0, 1, \dots, N-1 \text{ and}$$

$$b_i = b \binom{N}{i} p^i (1-p)^{N-i} \text{ for } i = 0, 1, \dots, N. \text{ The optimal control}$$

of linear system (7) for the auxiliary task (9) contains in follow

**Theorem 1.** If  $\det[a_i I + B^T G_{i+1} B] \neq 0$  for  $i = 0, 1, 2, \dots, N-1$

where  $I$  is an identity matrix of appropriate dimension and

$$G_i = b_i I + G_{i+1} - G_{i+1}^T B [a_i I + B^T G_{i+1} B]^{-1} B^T G_{i+1} \text{ and } G_N = b_N I \quad (10)$$

then the optimal control is

$$u_i^* = [a_i I + B^T G_{i+1} B]^{-1} B^T G_{i+1} (y_i - a) \quad (11)$$

and

$$\inf_{u \in U} E \left\{ \sum_{i=0}^{N-1} (a_i\|u_i\|^2 + b_i\|y_i - a\|^2) + b_N\|y_N - a\|^2 \right\} = W_0(y_0) \text{ where}$$

$$W_N(y_N) = (y_N - a)^T G_N (y_N - a) \quad (12)$$

$$W_i(y_i) = (y_i - a)^T G_i (y_i - a) + \sum_{j=i+1}^N \text{tr}(\sigma^T G_j \sigma) \quad (13)$$

**Remark 2.** We use formulas (10)-(13) for the linear quadratic control with deterministic horizon  $N$ . Suffice it to put, that the density of random horizon is  $P(\tau=i)=0$  for  $i=0, 1, \dots, N-1$  and  $P(\tau=N)=1$ , then we have  $a_0 = a_1 = \dots = a_{N-1} = a$ ,  $b_0 = b_1 = \dots = b_{N-1} = 0$  and  $b_N = b$ .

**Example 3.** We consider the control of linear system with state equation (7) and criterion (8). We assume that the random horizon  $\tau$  has a Bernoulli distribution with probability  $p=0.5$  and system can be controlled up to 10 times.

Let  $a=b=1$ ,  $B = \begin{bmatrix} 2.7 & 0.02 \\ 0.03 & 2.1 \end{bmatrix}$ ,  $\sigma = \begin{bmatrix} 1.2 & -0.33 \\ 0.15 & 1.7 \end{bmatrix}$  and we re-

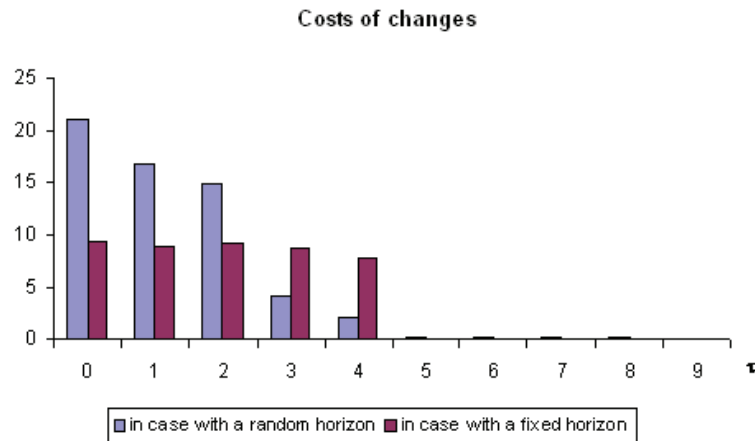
move the system e.g. from point  $y_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$  to  $a = \begin{pmatrix} 30 \\ 28 \end{pmatrix}$ .

To determine the optimal control with random horizon we must consider the auxiliary task (9). In case with fixed horizon we assume that the horizon is  $N = E\tau = 5$ .

The simulation shows the differences in these actions (changes, controls). In case with a fixed horizon the changes are gradual, evenly distributed over time. In case with an unknown horizon energy costs (outlays) are much higher at the beginning and smaller after the moment  $[E\tau]$ .

### 5. CONCLUSION

This paper shows how to model the changes of organization (structure) and how many expenditures must be incurred during the reorganization in case where the interval of these changes is not established. The ignorance of horizon introduces additional costs associated with the possibility of inheritance for every step.



**Figure 1** The possible costs of changes in cases with random and fixed horizons

The changes in random horizon for the initial phase should be much more distinct, more powerful, while for the later phase should be a small, residual, to absorb only external disturbances in the system. After exceeding (crossing) the moment  $[E\tau]$  we must eliminate some distur-

ances which generate the differences between Evader and Follower. Practically, until the moment  $[E\tau]$  the system (organization) must converge to the desired state.

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